SC-3801/100

SC-3801

M. Sc. (I.T.) (Sem. I) Examination

April / May - 2011

101 - Mathematics - I

Time : 3 Hours] [Total Marks : **70**

Instruction :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **M. Sc. (I.T.) (SEM. 1)**

Name of the Subject :
☛ **101 - MATHEMATICS - 1**

☛ Subject Code No. : 3 8 0 1 ☛ Section No. (1, 2,.....) : NIL

Seat No. :

Student's Signature

(2) Attempt **all** questions.

(3) Figures to the right indicate full marks.

(4) Follow usual notations.

(5) Use of non-programmable calculator is allowed.

1.(a) Define the converse relation. Let R be a relation on A. Let R^{-1} be the converse relation of R. Prove that the range of R = the domain of R^{-1}. (4)

OR

(a) Define: An equivalence class. Prove that any two equivalence classes are either identical or disjoint.

(b) Answer any three of the following. (9)

(i) Let A = {1,2,3,4,5}. Let D means divides be a relation on A. Find D and show that it is transitive.

(ii) Let I be the set of positive integers. Let $\equiv_m$ be the congruence modulo m relation on I. Show that it is symmetric and transitive.

(iii) Let P(A) be the power set of A. Let $\subseteq$ (inclusion) be a relation on P(A). Show that it is transitive but not symmetric.

(iv) Show that the union of two equivalence relations is not an equivalence relation.

(v) Let R be the set of real numbers. Define a binary operation $*$ on R as $a * b = a + b + 10$. Find the value of $(25*15)*12$.

(c) Define a function. Let $f : A \rightarrow B$ be a function. If $X, Y \subseteq A$ then prove that $f(X \cup Y) = f(X) \cup f(Y)$. (5)

OR

(c) Define the inverse of a function. Prove that the inverse of a one-to-one and onto function is also one-to-one and onto.

2.(a) Answer any two of the following. (6)

(i) Let f, g : R → R be two functions. Let f(x) = x+3 and (gof)(x) = 3x—5.Find the formula for the function g.

(ii) Let A = {1, 2, 3, 4, 5}. Let f, g, h : A→ A be functions defined by f = {(1, 2), (2,3), (3, 2), (4, 5), (5, 1)}; g = {(1, 3), (2, 2), (3, 1), (4, 1), (5, 4)} and h = { (1, 2), (2, 4), (3, 3), (4, 5), (5, 1)}. (i) Determine which of the functions f, g, h are onto. (ii) Find fo(hog).

(iii) Let f : R → R be a function defined as f(x) =(3x + 4). Prove that f is one–to–one and onto and hence find the inverse of f.

(iv) Consider the real valued functions $f(x) = 3x^2 + 4x - 5$ and g(x) = (5x-9). Find (fog)(x) and (gof)(5).

(b) Define any three of the following giving one illustration to each. (3)

(i) An upper triangular matrix. (ii) Transpose of a matrix. (iii) Non-Singular matrix. (iv)symmetric matrix.(v) Skew-Hermitian matrix.

(c) Answer any three of the following. (9)

(i) In the usual notations prove that $(\chi_{A\cup B})$ = Max. (χ_A, χ_B).

(ii) If $A = \begin{bmatrix} -2 & 4 & 5 \\ -3 & 1 & 0 \\ 2 & 6 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 & -2 \\ 3 & -2 & 1 \\ 5 & 4 & -1 \end{bmatrix}$ then find

(i) a matrix C such that 12A + 3C = 6B and (ii) a matrix BA.

(iii) Obtain the adjA of a matrix $A = \begin{bmatrix} 3 & 0 & 1 \\ 2 & -4 & 6 \\ 4 & -1 & 5 \end{bmatrix}$ and calculate A(adjA).

(iv) Find the inverse of the following matrix A using elementary row transformations.

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -3 & 6 & 2 \\ 4 & 5 & 2 \end{bmatrix}.$$

(v) Show that the every square matrix can be expressed as the sum of a symmetric matrix and a skew-symmetric matrix.

3.(a) State different measures of central tendency. Which measure will you consider ideal. Why? (4)

OR

(a) Explain Dispersion. Discuss the merits and demerits of a variance as a measure of dispersion.

(b) Answer any tw0 of the following. (10)

(i) Find mean, median, mode and P_{60} for the following data.

Age (in years)	30-34	35-39	40-44	45-49	50-54	55-59
No. of persons.	4	6	12	18	14	8

(ii) Calculate the harmonic mean, Q_3 and D_6 for the following data.

Class :	0-10	10-20	20-30	30-40	40-50	50-60
Frequency :	10	16	40	20	10	4

(iii) Find the coefficient of variance for the following data.

Wages:	>180	>190	>200	>210	>220	>230	>240
No. of workers:	40	36	28	13	4	1	0

(where > means greater than)

(c) Define: (i) Random experiment. (ii) Probability function. State and prove addition theorem on probability. (4)

OR

(c) Define: (i) Sample space. (ii) Conditional probability. State and prove multiplication theorem on probability.

4.(a) Answer any two of the following. (6)

(i) A box I contains 16 non-defective and 4 defective electric bulbs and box II contains 12 non-defective and 4 defective bulbs. If integer 1 or 2 appears on the face of the die, box I is selected and a bulb is drawn at random from it. If integer other than 1 and 2 appears on the die, box II is selected and a bulb is drawn from it. A bulb is drawn at random and it was found defective. What is the probability that it was selected from the box II ?

(ii) A card is selected from a pack of well shuffled 52 cards. Let A be the event that the selected card is of ace and B is the event that the card is of heart. Find P(A/B).

(iii) If A and B are independent events then prove that A^- and B are also independent.

(b) State the probability function of a binomial distribution with parameters n and p. Show that its mean is np and variance is npq, where q=1–p. (4)

OR

(b) State the probability function of a Poisson distribution with parameter m. Show that its variance is m.

(c) Answer any two of the following. (6)

(i) The mean and standard deviation of a random variable X are 10 and 5 respectively. Find E(4 X + 5) and V(3X + 5)

(ii) A multiple choice test consists of 8 questions with 3 answers to each question (of which only one is correct). A student answers each question by tossing a die and checking the first answer if he gets 1 or 2; the second answer if he gets 3 or 4 and third answer if he gets 5 or 6. To get a distinction, the student must secure at least 75% correct answer. If there is no negative marking, what is the probability that the student secures a distinction?

(iii) Between 10 and 11 a.m. the average number of phone calls per minute coming in to the switch board of a company is 2.5. Using Poisson distribution, find the probability that during one particular minute there will be (i) no phone call at all. (ii) exactly 3 calls.